

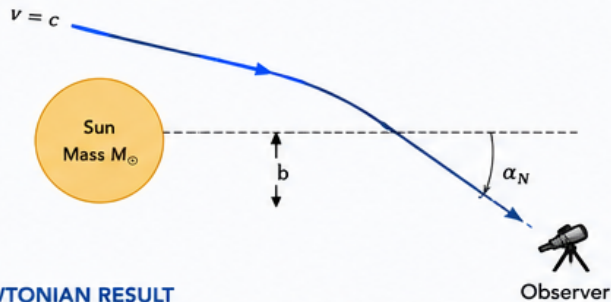
CALCULATIONS OF LIGHT DEFLECTION BY THE SUN

Three theoretical results for the total deflection angle α of a light ray grazing the Sun ($b = R_{\odot}$)

1. NEWTONIAN CALCULATION (CORPUSCULAR LIGHT)

Light is treated as particles of speed c moving in the Newtonian gravitational field of the Sun.

GEOMETRY



NEWTONIAN RESULT

$$\alpha_N = \frac{2GM_{\odot}}{c^2 b}$$

For grazing incidence, $b = R_{\odot}$
 $\alpha_N \approx 0.87$ arcseconds

DERIVATION SKETCH

Using inverse-square gravitational force and small-angle scattering of a particle passing the Sun with speed c .

$$F = \frac{GM_{\odot}m}{r^2} \rightarrow \text{transverse impulse} \rightarrow \text{deflection}$$

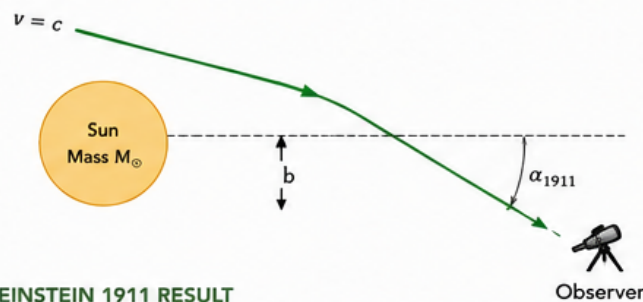
KEY ASSUMPTIONS

- Light is a stream of particles (corpuscles)
- Newtonian gravity: instantaneous action at a distance
- Space is Euclidean; time is absolute

2. EINSTEIN 1911 CALCULATION (EQUIVALENCE PRINCIPLE, "ENTWURF")

Light is treated as an electromagnetic wave. Using the equivalence principle in a static gravitational field.

GEOMETRY



EINSTEIN 1911 RESULT

$$\alpha_{1911} = \frac{2GM_{\odot}}{c^2 b}$$

For grazing incidence, $b = R_{\odot}$
 $\alpha_{1911} \approx 0.87$ arcseconds

METHOD (EQUIVALENCE PRINCIPLE)

In a uniform gravitational field g , a light ray in a medium of refractive index n experiences an apparent deflection equivalent to being in free fall.

This gives the same result as the Newtonian calculation:

$$\alpha_{1911} = \alpha_N$$

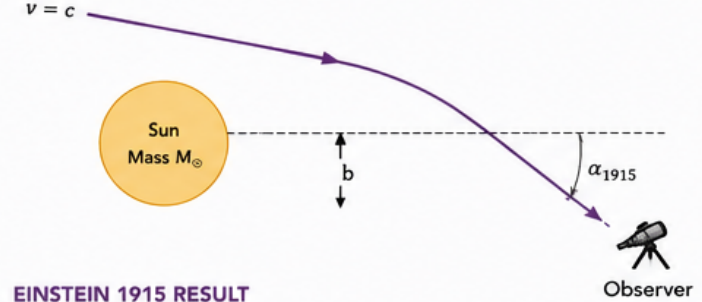
KEY ASSUMPTIONS

- Light is an electromagnetic wave
- Equivalence principle applied in the weak-field limit
- Space is Euclidean; gravitational field is static

3. EINSTEIN 1915 CALCULATION (GENERAL RELATIVITY)

Light follows null geodesics in curved space-time (predicted by General Relativity).

GEOMETRY



EINSTEIN 1915 RESULT

$$\alpha_{1915} = \frac{4GM_{\odot}}{c^2 b}$$

For grazing incidence, $b = R_{\odot}$
 $\alpha_{1915} \approx 1.75$ arcseconds

METHOD (GENERAL RELATIVITY)

Solve the geodesic equation for light in the Schwarzschild metric (weak-field limit). The deflection comes from two contributions:

$$\text{Curvature of space (spatial curvature)} \\ \frac{2GM_{\odot}}{c^2 b}$$

+

$$\text{Curvature of time (gravitational time dilation)} \\ \frac{2GM_{\odot}}{c^2 b}$$

$$\text{Total: } \alpha_{1915} = \frac{4GM_{\odot}}{c^2 b} = 2 \alpha_N = 2 \alpha_{1911}$$

KEY ASSUMPTIONS

- Light follows null geodesics in curved space-time
- Schwarzschild solution (spherically symmetric mass)
- Weak-field approximation (valid for Sun)

NUMERICAL COMPARISON (GRAZING THE SUN)

Theory	Formula for α ($b = R_{\odot}$)	Deflection
Newton (corpuscular)	$\frac{2GM_{\odot}}{c^2 R_{\odot}} \cdot c^2 R_{\odot}$	0.87 arcsec
Einstein 1911 (equivalence)	$\frac{2GM_{\odot}}{c^2 R_{\odot}} \cdot c^2 R_{\odot}$	0.87 arcsec
Einstein 1915 (GR)	$\frac{4GM_{\odot}}{c^2 R_{\odot}} \cdot c^2 R_{\odot}$	1.75 arcsec

CONSTANTS USED

$$\begin{aligned} G &= 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \\ M_{\odot} &= 1.9885 \times 10^{30} \text{ kg} \\ R_{\odot} &= 6.957 \times 10^8 \text{ m} \\ c &= 2.99792458 \times 10^8 \text{ m s}^{-1} \\ 1 \text{ arcsec} &= 4.84813681 \times 10^{-6} \text{ rad} \end{aligned}$$

HISTORICAL NOTE

The 1919 solar eclipse expeditions measured a deflection of about 1.6–1.98 arcseconds, consistent with the Einstein 1915 prediction (within experimental uncertainty), thereby providing the first observational confirmation of General Relativity.